A UNIFIED ANALYSIS FOR PLANAR TRANSMISSION LINES

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Summary

A unified analysis for planar transmission lines is presented, which needs about 2 orders of magnitude less computer time than the spectral domain method. It is applied to various microstrip-lines and to fin-lines.

Introduction

This contribution deals with a rigorous and efficient analysis of planar transmission lines and its application to fin-lines. Its main difference to existing methods (the spectral domain technique[1] or the method of autonomous multimodal blocks[2]) is a reduction in computer time of about two orders of magnitude. Hence, it should be well suited for a computer-aided design of microwave planar circuits.

Analysis

The structure which has been analyzed consists of an arbitrary number of metallic strips which are deposited on either side of a dielectric substrate. This planar circuit may be mounted either in the H-plane or in the E-plane of a rectangular box. Hence, the structure can be specialized to represent a microstrip line, coupled striplines, a slot line, a coplanar line, a microstrip line with tuning septums, a bilateral, unilateral, or antipodal fin-line, and a multi-slot fin-line. For explaining the calculation procedure, the cross-section of the latter is shown in Fig.1. The metallic strips are assumed to have finite thickness. This eliminates, on one hand, the existence of field singularities due to an edge condition while it is furthermore realistic at frequencies in the upper mm-wave range.

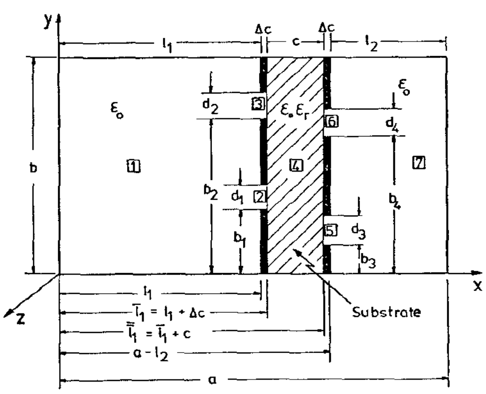

Fig.1 Cross-section of a general fin-line

The eigenmode analysis starts with the well-known mode-matching method, which shows some important advantages over other methods: Its final equations can be interpreted physically. This allows simplifications of great consequences as will be shown below. Moreover, the method can also directly be applied at the cutoff-frequency. An inherent disadvantage is, however, the poor convergence of the solutions. It was the main task of our investigations to remove this restriction.

In the analysis, the cross-section is divided into 7 regions with 8 boundaries separating these regions from one another. Due to the dielectric substrate, the field is hybrid and can be calculated from 2 scalar potential functions. These functions are written down for every sub-region and matched at the various interfaces. This yields enough relations to determine the unknown expansion coefficients. It turns out that the expansion coefficients of the slot regions and of the dielectric region can be eliminated and explicitly related to the expansion coefficients of sub-region 1 and 7. This yields the following homogeneous system of equations:

$$A_t f_t = \sum_p X_{p,1} F_1(t,p) \; ; \; G_t g_t = \sum_p X_{p,7} F_7(t,p) \; ; \; t = 0,1,2\ldots \qquad \text{(HE-modes)}, \; (1)$$

$$\overline{A}_t \overline{f}_t = \sum_p \overline{X}_{p,1} \overline{F}_1(t,p) \; ; \; \overline{G}_t \overline{g}_t = \sum_p \overline{X}_{p,7} \overline{F}_7(t,p) \; ; \; t = 1,2\ldots \qquad \text{(EH-modes)}, \; (2)$$

Here A_t, G_t, $\overline{A}_t$, $\overline{G}_t$ mean the expansion coefficients of the HE-modes and the EH-modes in sub-regions 1 and 7, respectively. X_t and $\overline{X}_t$ are linear combinations of these coefficients. f_t, g_t etc. contain the frequency dependence. The abbreviations F_1, F_7, $\overline{F}_1$, and $\overline{F}_7$ depend only on geometric parameters and not on frequency. They can be written as

$$F_1(t,p) = \sum_i \sum_s f_i(t,s)\, f_i(p,s)\, b/d_i = \sum_i F_s \; . \qquad (3)$$

Here b means waveguide height and d_i means width of the i-th slot. The function $f_i(p,s)$ belongs to the i-th slot. It is given by

$$f_i(p_1,p_2) = \frac{2}{p_1 \pi} \left[\frac{(-1)^{p_2} \cdot \sin(p_1 \pi (b_i + d_i)/b) - \sin(p_1 \pi \cdot b_i / b)}{1 - (p_2^2/p_1^2)(b^2/d_i^2)} \right] \qquad \text{if } p_1 d_i \neq p_2 b,$$

$$= \frac{d_i}{b} \cos(p_2 \pi\, b_i / d_i) \qquad \text{if } p_1 d_i = p_2 b,$$

$$f_i(0,0) = \frac{2 d_i}{b} \; . \qquad (4)$$

b_i means distance between slot and bottom wall of the waveguide housing. (p_1 and p_2 are integer.)

The computer time can be reduced by utilizing that F_s depends on the summation index s as shown in Fig.2. F_s monotonically increases up to a maximum value which is achieved at s_m. Differentiating (3) with respect to s yields

$$s_m = \sqrt{(p^2 + t^2)/2}\; d_i / b \; . \qquad (5)$$

For $s > s_m$, F_s strongly decreases and can be neglected when s exceeds $s_m + \Delta s$. Δs depends on the ratio of the slot width to the waveguide height.

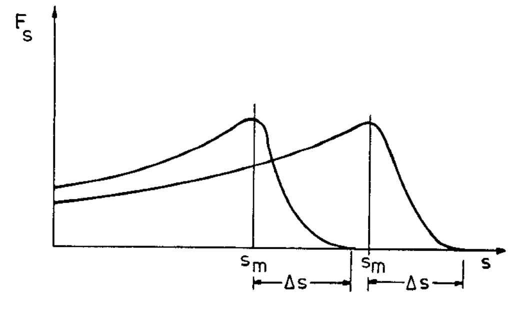

Fig.2 F_s of equ. (3) versus integer s; parameter are the integers p and t and the slot width d_i (equ. (5)).

The propagation constant is found by equating the determinant of the homogeneous system (1) and (2) to zero. As an example, numerical results are compared with published values in Table 1. The agreement is excellent.

f GHz	number of terms	k/k_z	k/k_z/3/
	6	0.3977	
5	10	0.3972	0.40
	15	0.3968	
	6	0.3731	
15	10	0.3725	0.38
	15	0.3721	

Table 1: Microstrip with tuning septum, comparison with Reference 3

Approximations

Calculation at cutoff. Up to now, the computer time is very large. Moreover, one must be very careful in order not to overlook one or the other zero of the determinant. Both restrictions can be removed. The first step is to replace the analysis by a calculation at the cut-off-frequency of every mode. Then the HE-modes are reduced to pure TE-modes and the EH-modes to pure TM-modes. Equations (1) and (2) are then decoupled and can separately be solved. Equating their determinants to zero yields the cutoff-frequencies. The propagation constants can be calculated from the cutoff-frequencies by utilizing the concept of an equivalent dielectric constant k_e, which has been defined in Reference 4. Our calculations have proven that the equivalent dielectric constant is nearly constant versus frequency as has been claimed[4] provided that the relative dielectric constant ε_r of the substrate material is only small. The substrate material, which is often used, is RT-duroid with $\varepsilon_r = 2.22$. In this case, k_e may be assumed to be constant. It can then be calculated from its definition[4] as the squared ratio of the cutoff wave number for $\varepsilon_r=1$ to that obtained for the actual ε_r.

The concept of an equivalent dielectric constant is justified by this feature alone. Moreover, the numerical calculations show that this quantity is of large practical importance even for arbitrary permittivities, because it depends on frequency linearly. This is proven in Fig.3 showing the equivalent dielectric constant of a unilateral fin-line versus frequency. Hence, it suffices in any case to calculate k_e twice (e.g. at cutoff and at the upper band edge). The dispersion relation can then easily be formulated and evaluated.

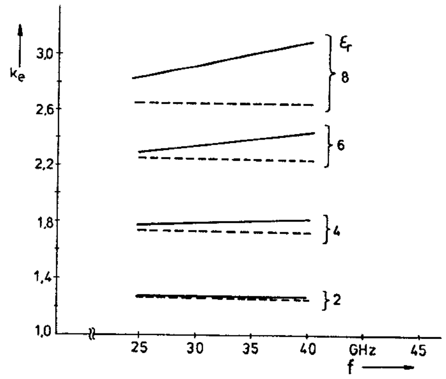

Fig.3 Equivalent dielectric constant of a unilateral fin-line versus frequency; parameter is the permittivity of the substrate.

A comparison between such approximations to the propagation constant and exact values shows that the assumption of a linearly frequency dependent equivalent dielectric constant is justified. This is illustrated by the results presented in Table 2.

$\frac{d_2}{b}$	f / k_z	25	40 GHz
0.0	approx.	0.519	0.890
	exact	0.523	0.897
0.3	approx.	0.391	0.803
	exact	0.392	0.809

Table 2: Comparison between approximate and exact propagation constants k_z of a unilateral fin-line with one ($d_2 = 0$) and two slots for different frequencies f ($d_1 = 0.1b$, $\varepsilon_r = 2.22$)

Relation between fundamental and higher-order modes. Using this approximation, the computer time is reduced by a factor of 1:N, with N being the number of interesting frequencies. The cutoff-frequency of every wanted eigenmode must, however, still be calculated. This can be avoided by the second step of our approximate method, which establishes a relation between all higher-order modes and the fundamental mode. This shall be demonstrated for the unilateral fin-line. Let us concentrate on the TE_{mn}-modes. Then (1) reduces to

$$A_t f_t = \sum_p X_{p,1} F_1(t,p) \ , \tag{6}$$

which can also be written in the following way:

$$A_t f_t = X_{t,1} \sum_p F_1(t,p) \ X_{p,1}/X_{t,1} \ . \tag{7}$$

Its solution reads

$$X_{t,1} = A_t f_t \ / \ \Sigma F_1(t,p) \ X_{p,1}/X_{t,1} \equiv K_t f_t \ . \tag{8}$$

It turns out that K_t is nearly constant with respect to index m and also with respect to index p or n, respectively. This has been proven numerically for all sets of parameters investigated. The physical explanation of this effect can also be given. It can namely be shown that K_t is proportional to the fringing field capacitance which is due to the metallic fins. This capacitance does not, however, depend on the mode number.

The calculation of the cutoff-frequencies can now be performed in the following way: The characteristic equation is exactly solved for the fundamental mode. Thus also K_t is fixed. Then (8) represents a closed-form solution for all higher-order modes. Thus the computer time has been reduced a second time by the ratio 2:M with M the number of the wanted eigenmodes. The total reduction in computer time is hence 2:NM, which is equivalent to a reduction by two orders of magnitude for most applications. The validity of these approximations is again illustrated by a comparison to exact values, which has been drawn in Table 3.

m,n	k_c(exact)	k_c(approx.)	
1,0	0.252	-	HE-modes
1,1	0.879	-	
3,0	0.998	1.002	
5,1	2.041	2.039	
1,3	2.658	2.646	
1,1	1.279	-	EH-modes
2,1	1.923	1.925	
1,2	1.986	-	
3,1	2.044	2.050	
3,2	2.545	2.535	
5,1	2.907	2.913	

Table 3: Comparison between exact and approximate cutoff wave numbers of a bilateral fin-line (WR-28 waveguide, $c = 0.254$ mm, $l = 3.4155$ mm, $d_1 = 0.56$ mm, $b_1 = 1.5$ mm, $\varepsilon_r = 2.22$)

Similar closed-form solutions can also be derived for frequencies above cutoff (if one wants to circumvent the concept of an equivalent dielectric constant). One must then simultaneously solve the equations for the HE- and the EH-modes.

Simplification for fundamental mode. The last step is a simplified calculation of the fundamental mode. It is based on the asymptotic behaviour of function F_s in (3). We will first regard the case that the slot width is much less than the waveguide height. Then the maximum of F_s is at $s_m < 1$ so that

$$F_1(t,p) \approx f_1'(t) f_1'(p). \qquad (9)$$

Equations (1) and (2) can now be solved approximately:

$$A_t f_t / f_1'(t) \approx \sum_p X_{p,1} f_1'(p) \;;\; G_t g_t / f_7'(t) \approx \sum_p X_{p,7} f_7'(p) \;. \qquad (10)$$

These relations can easily be evaluated for the expansion coefficients.

The method can also be used when the slot width is not much less than the waveguide height. Then one solves the k-th row of the matrix equation (1) for $X_{k,1}$ and inserts this expression into the other equations. This yields

$$A_t f_t = A_i f_i F_1(t,i)/F_1(i,i) + \sum_p X_{p,1} \left[\cdots \right] \;. \qquad (11)$$

The term in brackets is very small and can be neglected so that we have an explicit solution for A_t.

The validity of this last approximation is confirmed by a comparison between our approximate values for the propagation constant of the fundamental mode of a unilateral fin-line and values derived in Reference 5 by using the spectral domain method. The results given in Table 4 show a good agreement. (In fact, the effective dielectric constant has been shown which is defined as the squared ratio between the propagation constant and the wave number.)

d \ D	0.2	0.8	1.6 mm	
0.1	1.129	1.152	1.165	this theory
	1.13	1.15	1.17	Ref. 5
0.25 mm	1.061	1.066	1.074	this theory
	1.06	1.06	1.07	Ref. 5

Table 4: Comparison between this theory and Reference 5 for $\varepsilon_{eff} = (k_z^2/k^2)$ of a unilateral fin-line at 33 GHz; slot width d, slot height D, WR-28 waveguide, RT-duroid 5880.

In conclusion, it can be stated that the eigenmode analysis of a planar structure has been simplified considerably. It can now be performed on a desk computer. The introduced error has been proven to be less than 1%.

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